Temperature Based Identification of Thermal Properties of a One Dimensional Transient Convection Model of a Slender Cylindrical Fin

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Introduction

- There is currently no simple method for determining the thermal conductivity of a material.
- The ASTM standards for measuring the thermal conductivity coefficient require expensive equipment and precise setup making the setup and results difficult to replicate [1,2].
- The proposed approach promises to be easier to implement than the current ASTM standards and therefore find broader applications in additive manufacturing environments.
- For this study, a one-dimensional transient heat diffusion PDE with a closed-form solution is used to model the slender test coupons.
- The heat transfer coefficient and thermal conductivity of the rod were estimated using a modified Levenberg-Marquardt (LM) nonlinear least squares algorithm.

Experimental Setup

- The boundary temperature is regulated by a PID controller running on an Arduino MEGA 2560.
- Additional thermocouples are embedded along the length of the rod.
- The heating element consists of insulated NiCr wire tightly wrapped around one end of the rod which is used to heat the boundary to the desired temperature.
- Forced convection is provided by a small wind tunnel that administers uniform, laminar flow across the rod.

Results

Possible errors in parameter estimation can be attributed to:
- Surface roughness
- Thermocouple placement
- Variation in thermal conductivity at different temperatures
- Turbulent flow

Governing Equation

\[
\frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial x} - v \theta = 0
\]

Where,
- \( k \) = the Perimeter
- \( v = \frac{hs}{\rho A} \) = the Cross-Sectional Area

Boundary Conditions: \( \frac{\partial \theta}{\partial x} (0, t) = -\frac{P(1 - e^{-\alpha t})}{kA} \), \( \frac{\partial \theta}{\partial x} (L, t) = 0 \)

Solution to the Governing Equation:

\[
\theta(x, t) = P \left( e^{-\alpha t} - e^{-\alpha L} \right) \left( 1 + e^{-\alpha t} \right) \frac{kA}{\nu L} \theta
\]

\[+ 2 P \sum_{i=1}^{n} \left( e^{-\beta (x + \nu T)^2} - e^{-\beta x^2} \right) \left( 1 + e^{-\beta (x + \nu T)^2} \right) \cos \frac{\beta x}{\nu L} \]

Where,
- \( P \) = the Power into the Rod
- \( A \) = the Area of the Rod
- \( h \) = the Heat Transfer Coefficient
- \( k \) = the Thermal Conductivity
- \( \alpha \) = the Reduced Temperature
- \( \nu \) = the Reduced Time

Conclusions

- The four unknown parameters in the model are the heat transfer coefficient, thermal conductivity, power, and the delay constant.
- The LM algorithm seeks the “best” parameters which minimize the sum of squared errors between the model-predicted and experimentally measured temperatures at each thermocouple location at each instant in time [5].

- Rods made of copper, aluminum, and stainless steel are used for the experiment.
- Each has a published thermal conductivity which was compared to the estimated values.

References


