Temperature Based Identification of Thermal Properties of a One Dimensional Transient Convection Model of a Slender Cylindrical Fin

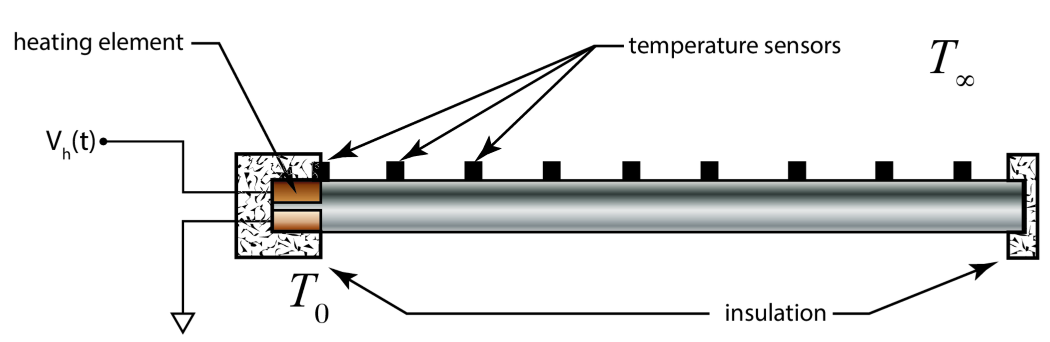
Lauren Tomanek, Graduate Student, Dr. Daniel S. Stutts, Associate Professor
Department of Mechanical and Aerospace Engineering
Missouri S&T University of Science & Technology

Introduction

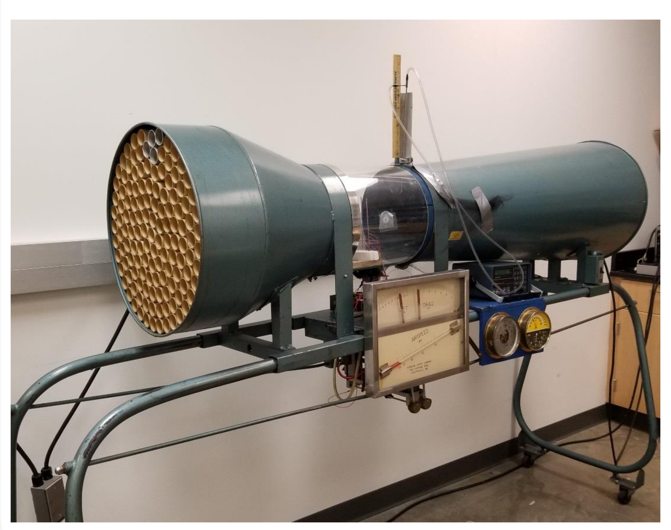
- There is currently no simple method for determining the thermal conductivity of a material.
- The ASTM standards for measuring the thermal conductivity coefficient require expensive equipment and precise setup making the setup and results difficult to replicate [1,2].
- The proposed approach promises to be easier to implement than the current ASTM standards and therefore find broader applications in additive manufacturing environments.
- For this study, a one-dimensional transient heat diffusion PDE with a closed-form solution is used to model the slender test coupons.
- The heat transfer coefficient and thermal conductivity of the rod were estimated using a modified Levenberg-Marquardt (LM) nonlinear least squares algorithm.

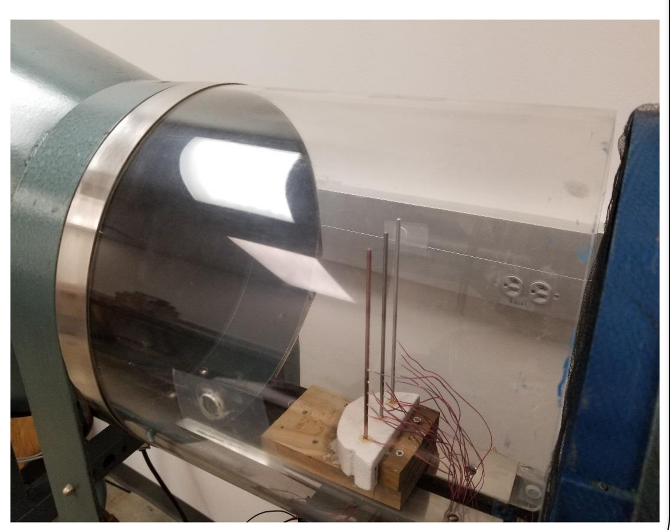
Experimental Setup

- The boundary temperature is regulated by a PID controller running on an Arduino MEGA 2560.
- Additional thermocouples are embedded along the length of the rod.
- The heating element consists of insulated NiCr wire tightly wrapped around one end of the rod which is used to heat the boundary to the desired temperature.



- Forced convection is provided by a small wind tunnel that administers uniform, laminar flow across the rod.





Governing Equation

Governing Equation [3,4]:

$$\frac{\partial \theta}{\partial t} = \kappa \frac{\partial^2 \theta}{\partial x^2} - \nu\theta$$

Where,

$$\kappa = \frac{k}{\rho c} \qquad \nu = \frac{hs}{\rho A\, c}$$

Boundary Conditions:

$$\frac{\partial \theta}{\partial x}(0,t) = \frac{-P\left(1 - e^{-\alpha(t+\tau)}\right)}{kA} \qquad \frac{\partial \theta}{\partial x}(L,t) = 0$$

Solution to the Governing Equation:

$$\theta(x,t) = \frac{P\kappa\left(\nu e^{-\alpha\tau - \nu t} - \nu e^{-\alpha(t+\tau)} + (\nu-\alpha)\,(1-e^{-\nu t})\right)}{\nu L k A\,(\nu-\alpha)}$$

$$+ \frac{2P\kappa}{LkA}\sum_{n=1}^{\infty}\left[\left(\frac{e^{-(\beta^2+\nu)t-\alpha\tau} - e^{-\alpha(t+\tau)}}{\beta^2-\alpha+\nu} + \frac{1}{\beta^2+\nu} - \frac{(\beta^2-\alpha+\nu)e^{-(\beta^2+\nu)t}}{(\beta^2-\alpha+\nu)(\beta^2+\nu)}\right)\cos\frac{\beta}{\sqrt{\kappa}}x\right]$$

Where,

$$\beta = \frac{n\pi}{L}\sqrt{\kappa}$$

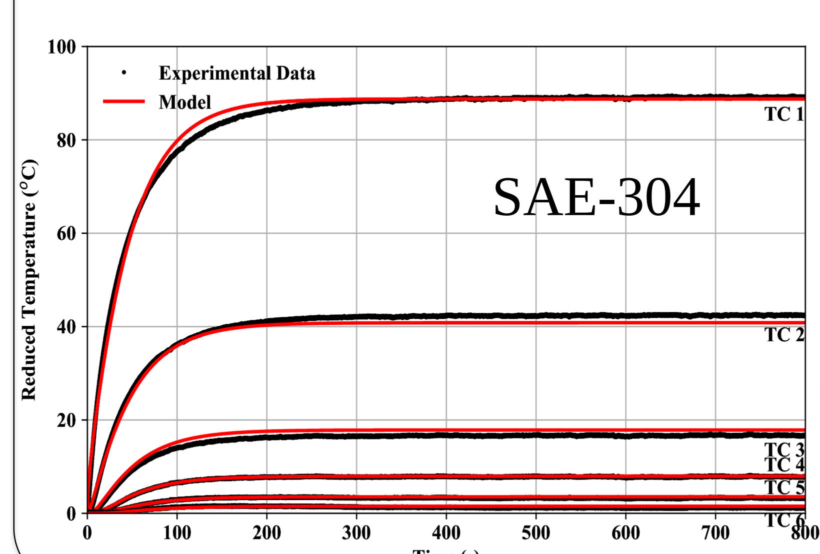
c	Specific Heat
h	Heat Transfer Coefficient
s	Perimeter
A	Cross-Sectional Area
L	Length of the Rod
P	Power into the Rod
α	Heat Flow into the Boundary
P	Density
τ	Thermocouple Delay
T_∞	Ambient Temperature
$\theta(x,t)$	Reduced Temperature

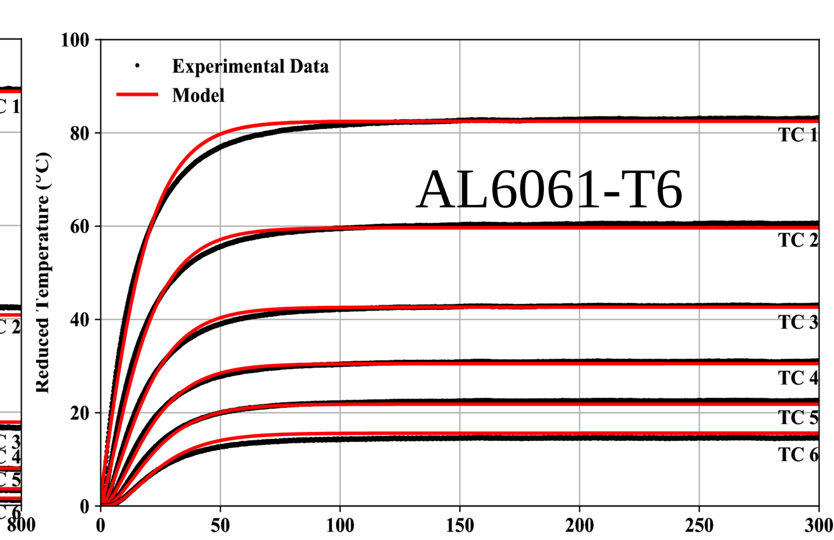
Results

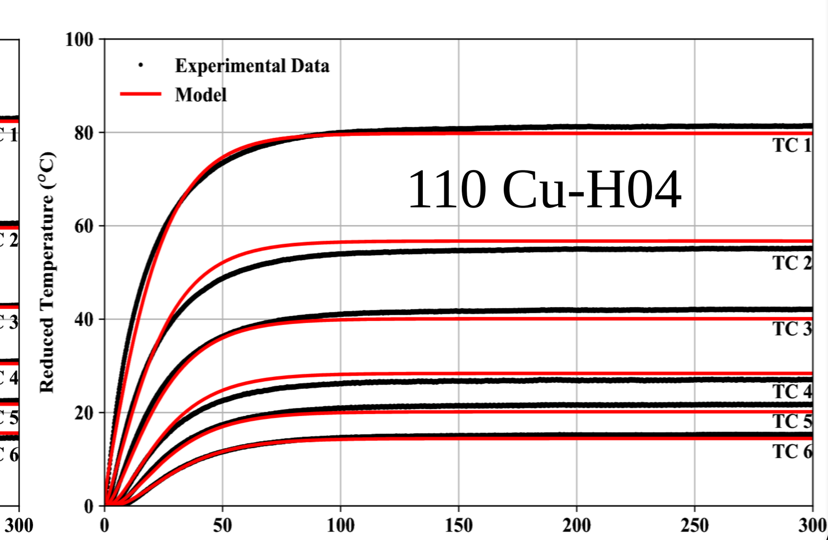
Possible errors in parameter estimation can be attributed to:

- Surface roughness
- Thermocouple placement
- Variation in thermal conductivity at different temperatures
- Turbulent flow

Material and Standard Thermal	Parameter	Units	50°C	75°C	100°C
SAE-304:	k	W/mK	15.71 ± 0.25	14.86 ± 0.12	15.70 ± 0.11
	h	W/m^2K	192.2 ± 3.1	165.4 ± 1.3	170.4 ± 1.1
16.2 W/mK	Mean Standard Error	°C	0.514	0.672	0.796
	Percent Difference	%	-3.020	-8.295	-3.086
AL6061-T6:	k	W/mK	151.3 ± 1.3	156.2 ± 1.0	152.9 ± 0.8
	h	W/m^2K	144.3 ± 1.2	141.2 ± 0.9	139.2 ± 0.8
167 W/mK	Mean Standard Error	°C	0.534	0.713	0.849
	Percent Difference	%	-9.381	-6.479	-8.439
110 Cu-H04:	k	W/mK	380.5 ± 4.5	387.3 ± 3.5	386.4 ± 2.8
	h	W/m^2K	164.5 ± 1.9	165.4 ± 1.5	166.3 ± 1.2
390 W/mK	Mean Standard Error	°C	0.722	0.956	1.153
	Percent Difference	%	-2.427	-0.703	-0.927







Background

- The four unknown parameters in the model are the heat transfer coefficient, thermal conductivity, power, and the delay constant.
- The LM algorithm seeks the "best" parameters which minimize the sum of squared errors between the model-predicted and experimentally measured temperatures at each thermocouple location at each instant in time [5].

$$E = \sum_{i=1}^{N_{tc}}\sum_{j=1}^{N}\left(\theta^{e}_{ij} - \theta(x_i, t_j)\right)^2$$

- Rods made of copper, aluminum, and stainless steel are used for the experiment.
- Each has a published thermal conductivity which was compared to the estimated values.

Conclusions

- The thermal conductivity and heat transfer coefficients which are parameters in a one-dimensional heat transfer model were estimated reasonably accurately.
- The model fit was better with less conductive metals
- The overall error was low for all materials and gave repeatable heat transfer coefficient values.
- This method will be useful in assessing the build quality and consistency of an additive manufacturing technique, and can provide a quick method for testing the thermal properties of a material and can be used for characterization.

References

[1] ASTM Standard C177-13, "Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded-Hot-Plate Apparatus." ASTM International, West Conshohocken, PA (2013). doi: 10.1520/c0177-13.
[2] ASTM Standard C518-17, "Test Method for Steady-State Thermal Transmission Properties by Means of the Heat Flow Meter Apparatus." ASTM International, West Conshohocken, PA (2017). doi: 10.1520/c0518-17.
[3] D. W. Mueller and H. I. Abu-Mulaweh, "Heat-Transfer Coefficient for a Long Fin Cooled by Convection and Radiation," Journal of Thermophysics and Heat Transfer, vol. 19, no. 4, pp. 583–586, Oct. 2005. doi: 10.2514/1.14730.
[4] H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Clarendon.
[5] D. W. Marquardt, "An Algorithm for Least-Squares Estimation of Nonlinear Parameters," Journal of the Society for Industrial and Applied Mathematics, vol. 11, no. 2, pp. 431–441, Jun. 1963. doi: 10.1137/0111030.